Almost Gorenstein rings

–towards a stratification of Cohen-Macaulay rings–

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based on the works jointly with

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Introduction

Why are there so many Cohen-Macaulay rings which are not Gorenstein?

Example 1.1 (Determinantal rings)

Let $S = k[X_{ij} \mid 1 \leq i \leq m, 1 \leq j \leq n\ ]$ $(2 \leq m \leq n)$ be the polynomial ring over a field k and put

$$R = S/\mathrm{I}_t(X)$$

where $2 \leq t \leq m$, $\mathrm{I}_t(X)$ is the ideal of S generated by $t \times t$-minors of $X = [X_{ij}]$.

Then

$$R \text{ is a Gorenstein ring} \quad \Longleftrightarrow \quad m = n.$$

Aim of this research

Find a new class of Cohen-Macaulay rings which may not be Gorenstein, but sufficiently good next to Gorenstein rings.

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Almost Gorenstein rings

History of almost Gorenstein rings

- [Barucci-Fröberg, 1997]

  $\cdots$ one-dimensional analytically unramified local rings

- [Goto-Matsuoka-Phuong, 2013]

  $\cdots$ one-dimensional Cohen-Macaulay local rings

- [Goto-Takahashi-T, 2015]

  $\cdots$ higher-dimensional Cohen-Macaulay local/graded rings

Survey on one-dimensional almost Gorenstein local rings

Setting 2.1

- $(R, \mathfrak{m})$ a CM local ring with $\dim R = 1$
- $|R/\mathfrak{m}| = \infty$
- $\exists\ \mathrm{K}_R$ the canonical module of R
- $\exists\ I \subsetneq R$ an ideal of R s.t. $I \cong \mathrm{K}_R$

Therefore, $\exists\ \mathrm{e}_0(I) > 0,\ \mathrm{e}_1(I) \in \mathbb{Z}$ s.t.

$$\ell_R(R/I^{n+1}) = \mathrm{e}_0(I)\binom{n+1}{1} - \mathrm{e}_1(I)$$

for $\forall n \gg 0$.

Set $\mathrm{r}(R) = \ell_R(\mathrm{Ext}_R^1(R/\mathfrak{m}, R))$.

Definition 2.2 (Goto-Matsuoka-Phuong)

We say that R is an *almost Gorenstein local ring*, if $\mathrm{e}_1(I) \le \mathrm{r}(R)$.

Suppose that I contains a parameter ideal $Q = (a)$ as a reduction, i.e.

$$I^{r+1} = QI^r \quad \text{for} \quad \exists\, r \ge 0.$$

We set

$$K = \frac{I}{a} = \left\{ \frac{x}{a} \mid x \in I \right\} \subseteq \mathrm{Q}(R).$$

Then K is a fractional ideal of R s.t.

$$R \subseteq K \subseteq \overline{R} \quad \text{and} \quad K \cong \mathrm{K}_R\,.$$

Theorem 2.3 (Goto-Matsuoka-Phuong)

R is an almost Gorenstein local ring $\Longleftrightarrow$ $\mathfrak{m}K \subseteq R$ (*i.e.* $\mathfrak{m}I \subseteq Q$)

Example 2.4

(1) $k[[t^3, t^4, t^5]]$

(2) $k[[t^e, t^{e+1}, \dots, t^{2e-3}, t^{2e-1}]]$ $(e \geq 4)$

(3) $k[[X, Y, Z]]/(X, Y) \cap (Y, Z) \cap (Z, X)$

(4) $k[[X, Y, Z, U, V, W]]/I$, where

$$\begin{aligned} I &= (X^3 - Z^2, Y^2 - ZX) + (U, V, W)^2 \\ &\quad + (YU - XV, ZU - XW, ZU - YV, ZV - YW, X^2U - ZW) \end{aligned}$$

Corollary 2.5 (Goto-Matsuoka-Phuong)

Suppose that R is complete and contains a field of ch $p > 0$. If R is F-pure, then R is an almost Gorenstein local ring.

Almost Gorenstein local rings of higher dimension

Setting 2.6

- $(R, \mathfrak{m})$ a CM local ring with $d = \dim R$
- $|R/\mathfrak{m}| = \infty$
- $\exists$ K_R the canonical module of R

Definition 2.7 (Goto-Takahashi-T)

We say that R is an *almost Gorenstein local ring* (abbr. AGL ring), if $\exists$ an exact sequence

$$0 \to R \to \mathrm{K}_R \to C \to 0$$

of R-modules s.t. $\mu_R(C) = \mathrm{e}^0_{\mathfrak{m}}(C)$.

Look at an exact sequence

$$0 \to R \to \mathrm{K}_R \to C \to 0$$

of R-modules. If $C \neq (0)$, then C is CM with $\dim_R C = d-1$.

Set $R_1 = R/[(0) :_R C]$.

Then $\exists\ f_1, f_2, \dots, f_{d-1} \in \mathfrak{m}$ s.t. $(f_1, f_2, \dots, f_{d-1})R_1$ forms a minimal reduction of $\mathfrak{m}_1 = \mathfrak{m}R_1$. Therefore

$$\mathrm{e}^0_{\mathfrak{m}}(C) = \mathrm{e}^0_{\mathfrak{m}_1}(C) = \ell_R(C/(f_1, f_2, \dots, f_{d-1})C) \geq \ell_R(C/\mathfrak{m}C) = \mu_R(C).$$

Thus

$$\mu_R(C) = \mathrm{e}^0_{\mathfrak{m}}(C) \Longleftrightarrow \mathfrak{m}C = (f_1, f_2, \dots, f_{d-1})C.$$

Hence, C is a *maximally generated maximal Cohen-Macaulay R_1-module* in the sense of B. Ulrich, which is called *an Ulrich R-module*.

Definition 2.8

We say that R is an *AGL ring*, if $\exists$ an exact sequence

$$0 \to R \to \mathrm{K}_R \to C \to 0$$

of R-modules s.t. either $C = (0)$ or $C \neq (0)$ and C is an Ulrich R-module.

Remark 2.9

Suppose that $d = 1$. Then TFAE.

(1) R is an AGL ring in the sense of Definition 2.8.

(2) R is an AGL ring in the sense of [GMP, Definition 3.1].

Theorem 2.10 (NZD characterization)

(1) *If R is a non-Gorenstein AGL ring of dimension $d > 1$, then so is $R/(f)$ for genaral NZD $f \in \mathfrak{m} \setminus \mathfrak{m}^2$.*

(2) *Let $f \in \mathfrak{m}$ be a NZD on R. If $R/(f)$ is an AGL ring, then so is R. When this is the case, $f \notin \mathfrak{m}^2$, if R is not Gorenstein.*

Corollary 2.11

Suppose that $d > 0$. If $R/(f)$ is an AGL ring for every NZD $f \in \mathfrak{m}$, then R is Gorenstein.

Example 2.12

Let $S = k[[X_{ij} \mid 1 \le i \le m, 1 \le j \le n\ ]]$ $(2 \le m \le n)$ and k an infinite field. We put

$$R = S/\mathrm{I}_t(X)$$

where $2 \le t \le m$, $X = [X_{ij}]$.

Then

$$R \text{ is an AGL ring} \iff m = n, \text{ or } m \ne n,\ t = m = 2$$

Proof

($\Leftarrow$) Let $X = \begin{pmatrix} X_1 & X_2 & \cdots & X_n \\ Y_1 & Y_2 & \cdots & Y_n \end{pmatrix}$ and put $R = S/\mathrm{I}_2(X)$ $(n \geq 2)$. Then R is a CM ring with $\dim R = n+1$ and $\mathrm{r}(R) = n-1$.

Notice that $\{X_i - Y_{i-1}\}_{1 \leq i \leq n}$ (here $Y_0 = Y_n$) forms a regular sequence on R and we get

$$R/(X_i - Y_{i-1} \mid 1 \leq i \leq n)R \cong k[[X_1, X_2, \ldots, X_n]]/\mathrm{I}_2(\mathbb{N}) = A$$

where $\mathbb{N} = \begin{pmatrix} X_1 & X_2 & \cdots & X_{n-1} & X_n \\ X_2 & X_3 & \cdots & X_n & X_1 \end{pmatrix}$. Then A is a CM ring with $\dim A = 1$ s.t.

$$\mathfrak{n}^2 = x_1\mathfrak{n} \text{ and } \mathrm{K}_A \cong (x_1, x_2, \ldots, x_{n-1}).$$

Thus A is an AGL ring, since $\mathfrak{n}(x_1, x_2, \ldots, x_{n-1}) \subseteq (x_1)$. Hence, R is AGL.

($\Rightarrow$) Use the minimal free resolution. □

Theorem 2.13

Let $(S, \mathfrak{n})$ be a Noetherian local ring, $\varphi : R \to S$ a flat local homomorphism. Suppose that $S/\mathfrak{m}S$ is a RLR. Then

$$R \text{ is an AGL ring} \iff S \text{ is an AGL ring.}$$

Therefore

- R is an AGL ring $\iff$ $R[[X_1, X_2, \ldots, X_n]]$ is an AGL ring.
- R is an AGL ring $\iff$ $\widehat{R}$ is an AGL ring.

Theorem 2.14

Suppose that $d > 0$. Let $\mathfrak{p} \in \operatorname{Spec} R$ and assume that $R/\mathfrak{p}$ is a RLR of dimension $d-1$. Then TFAE.

(1) $R \ltimes \mathfrak{p}$ *is an AGL ring.*

(2) R *is an AGL ring.*

Corollary 2.15 (Goto-Matsuoka-Phuong, Goto-Isobe-T)

Suppose that $d = 1$. Then TFAE.

(1) $R \ltimes \mathfrak{m}$ *is an AGL ring.*

(2) R *is an AGL ring.*

(3) $R \times_{R/\mathfrak{m}} R$ *is an AGL ring.*

Example 2.16

Let k be an infinite field. We consider

$$A_1 = k[[X,Y,Z,U,V,W]]/I, \quad A_2 = k[[X,Y,Z,U,V,W]]/J,$$

where

$$\begin{aligned} I &= (X^3 - Z^2, Y^2 - ZX) + (U,V,W)^2 \\ &\quad + (YU - XV, ZU - XW, ZU - YV, ZV - YW, X^2U - ZW) \end{aligned}$$

$$\begin{aligned} J &= (X^3 - Z^2, Y^2 - ZX) + (U^3 - W^2, V^2 - UW) \\ &\quad + (X,Y,Z)(U,V,W). \end{aligned}$$

Then

$$A_1 \cong k[[t^4,t^5,t^6]] \ltimes (t^4,t^5,t^6), \quad A_2 \cong k[[t^4,t^5,t^6]] \times_k k[[t^4,t^5,t^6]]$$

and hence A_1, A_2 are AGL rings.

Theorem 2.17

Let $(R, \mathfrak{m})$ be a CM complete local ring with $\dim R = 1$ and assume that $R/\mathfrak{m}$ is algebraically closed of characteristic 0. Suppose that R has finite CM representation type. Then R is an AGL ring.

Theorem 2.18 (Goto)

Suppose that R is a non-Gorenstein AGL ring with $\dim R \geq 1$. Let M be a finitely generated R-module. If

$$\operatorname{Ext}_R^i(M, R) = (0)$$

for $\forall i \gg 0$, then $\operatorname{pd}_R M < \infty$.

Corollary 2.19

Suppose that R is an AGL ring with $\dim R \geq 1$. If R is not a Gorenstein ring, then R is G-regular, i.e.

$$\operatorname{Gdim}_R M = \operatorname{pd}_R M$$

for every finitely generated R-module M.

Semi-Gorenstein local rings

In this section we maintain Setting 2.6.

Let $\mathcal{F} = \{I_n\}_{n \in \mathbb{Z}}$ be a filtration of ideals of R s.t. $I_0 = R$, $I_1 \neq R$.

We consider the R-algebras

$$\mathcal{R} = \sum_{n \geq 0} I_n t^n \subseteq R[t], \quad \mathcal{R}' = \sum_{n \in \mathbb{Z}} I_n t^n \subseteq R[t, t^{-1}], \quad \text{and} \quad G = \mathcal{R}'/t^{-1}\mathcal{R}'$$

associated to $\mathcal{F}$, where t is an indeterminate.

Let N denote the graded maximal ideal of $\mathcal{R}'$.

Theorem 3.1

Suppose that $\mathcal{R}$ is a Noetherian ring. If G_N is an AGL ring and $\mathrm{r}(G_N) \le 2$, then R is an AGL ring.

Proof.

We may assume $\mathrm{r}(G_N) = 2$. Since $\mathcal{R}'_N$ is an AGL ring with $\mathrm{r}(\mathcal{R}'_N) = 2$, we have

$$0 \to \mathcal{R}'_N \to \mathrm{K}_{(\mathcal{R}'_N)} \to C \to 0$$

where $C \cong$ a RLR of dim d.

Let $\mathfrak{p} = \mathfrak{m}R[t, t^{-1}]$ and set $P = \mathfrak{p} \cap \mathcal{R}'$. Then $P \subseteq N$, so that $R[t, t^{-1}]_{\mathfrak{p}}$ is an AGL ring, because

$$R[t, t^{-1}]_{\mathfrak{p}} = \mathcal{R}'_P = (\mathcal{R}'_N)_{P\mathcal{R}'_N}.$$

Hence, R is an AGL ring, since $R \to R[t, t^{-1}] \to R[t, t^{-1}]_{\mathfrak{p}}$ is flat. □

Example 3.2 (Barucci-Dobbs-Fontana)

Let $R = k[[x^4, x^6 + x^7, x^{10}]] \subseteq V$, where $V = k[[x]]$ and k is an infinite field of $\mathrm{ch}\, k \neq 2$. Let

- $H = \{v(a) \mid 0 \neq a \in R\}$ the value semigroup of R
- $\mathcal{F} = \{(xV)^n \cap R\}_{n \in \mathbb{Z}}$ the filtration of ideals of R.

Then

(1) $H = \langle 4, 6, 11, 13 \rangle$.

(2) $G \cong k[x^4, x^6, x^{11}, x^{13}]$ and G_N is an AGL ring with $\mathrm{r}(G_N) = 3$, so that $\mathcal{R}'_N$ is AGL.

(3) R is NOT an AGL ring and $\mathrm{r}(R) = 2$.

Therefore, $(\mathcal{R}'_N)_{\mathcal{P}\mathcal{R}'_N}$ is NOT an AGL ring.

Definition 3.3

We say that R is a *semi-Gorenstein local ring*, if R is an AGL ring which possesses an exact sequence

$$0 \to R \to \mathrm{K}_R \to C \to 0$$

s.t. either $C = (0)$, or C is an Ulrich R-module and $C = \oplus_{i=1}^{\ell} C_i$ for some cyclic R-submodule C_i of C.

Therefore, if $C \neq (0)$, then

$$C_i \cong R/\mathfrak{p}_i \quad \text{for } \exists\ \mathfrak{p}_i \in \operatorname{Spec} R$$

such that $R/\mathfrak{p}_i$ is a RLR of dimension $d-1$.

Notice that

- AGL ring with $\dim R = 1$
- AGL ring with $\mathrm{r}(R) \leq 2$

are semi-Gorenstein rings.

Proposition 3.4

Let R be a semi-Gorenstein local ring. Then $R_\mathfrak{p}$ is semi-Gorenstein for $\forall \mathfrak{p} \in \operatorname{Spec} R$.

Therefore, if $C \neq (0)$, then

$$C_i \cong R/\mathfrak{p}_i \quad \text{for } \exists\ \mathfrak{p}_i \in \operatorname{Spec} R$$

such that $R/\mathfrak{p}_i$ is a RLR of dimension $d-1$.

Notice that

- AGL ring with $\dim R = 1$
- AGL ring with $\mathrm{r}(R) \leq 2$

are semi-Gorenstein rings.

Proposition 3.4

Let R be a semi-Gorenstein local ring. Then $R_\mathfrak{p}$ is semi-Gorenstein for $\forall \mathfrak{p} \in \operatorname{Spec} R$.

Theorem 3.5

Let $(S, \mathfrak{n})$ be a RLR, $\mathfrak{a} \subsetneq S$ an ideal of S, $n = \mathrm{ht}_S \mathfrak{a}$. Let $R = S/\mathfrak{a}$. Then TFAE.

(1) *R is a semi-Gorenstein local ring, but not Gorenstein.*

(2) *R is CM, $n \geq 2$, $r = \mathrm{r}(R) \geq 2$, and R has a minimal S-free resolution:*

$$0 \to F_n = S^r \overset{\mathbb{M}}{\to} F_{n-1} = S^q \to F_{n-2} \to \cdots \to F_1 \to F_0 = S \to R \to 0$$

where

$$ {}^t\mathbb{M} = \begin{pmatrix} y_{21}y_{22}\cdots y_{2\ell} & y_{31}y_{32}\cdots y_{3\ell} & \cdots & y_{r1}y_{r2}\cdots y_{r\ell} & z_1 z_2 \cdots z_m \\ x_{21}x_{22}\cdots x_{2\ell} & 0 & 0 & 0 & 0 \\ 0 & x_{31}x_{32}\cdots x_{3\ell} & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & x_{r1}x_{r2}\cdots x_{r\ell} & 0 \end{pmatrix},$$

$\ell = n+1$, $q \geq (r-1)\ell$, $m = q-(r-1)\ell$, and $x_{i1}, x_{i2}, \ldots, x_{i\ell}$ is a part of a regular system of parameters of S for $2 \leq \forall i \leq r$.

When this is the case

$$\mathfrak{a} = (z_1, z_2, \ldots, z_m) + \sum_{i=2}^{r} \mathrm{I}_2 \begin{pmatrix} y_{i1} & y_{i2} & \cdots & y_{i\ell} \\ x_{i1} & y_{i2} & \cdots & x_{i\ell} \end{pmatrix}.$$

Example 3.6

Let $\varphi : S = k[[X, Y, Z, W]] \longrightarrow R = k[[t^5, t^6, t^7, t^9]]$ be the k-algebra map defined by

$$\varphi(X) = t^5,\ \varphi(Y) = t^6,\ \varphi(Z) = t^7 \text{ and } \ \varphi(W) = t^9.$$

Then

$$0 \to S^2 \xrightarrow{\mathbb{M}} S^6 \to S^5 \to S \to R \to 0,$$

where

$${}^t\mathbb{M} = \begin{pmatrix} W & X^2 & XY & YZ & Y^2-XZ & Z^2-XW \\ X & Y & Z & W & 0 & 0 \end{pmatrix}.$$

Hence R is semi-Gorenstein with $\mathrm{r}(R) = 2$ and

$$\operatorname{Ker}\varphi = (Y^2 - XZ, Z^2 - XW) + \mathrm{I}_2\begin{pmatrix} W & X^2 & XY & YZ \\ X & Y & Z & W \end{pmatrix}.$$

Almost Gorenstein graded rings

Setting 4.1

- $R = \bigoplus_{n \geq 0} R_n$ a CM graded ring with $d = \dim R$
- $(R_0, \mathfrak{m})$ a Noetherian local ring
- $|R_0/\mathfrak{m}| = \infty$
- $\exists\ \mathrm{K}_R$ the graded canonical module of R
- $M = \mathfrak{m}R + R_+$
- $a = \mathrm{a}(R) := -\min\{n \in \mathbb{Z} \mid [\mathrm{K}_R]_n \neq (0)\}$

Definition 4.2

We say that R is an *almost Gorenstein graded ring* (abbr. AGG ring), if $\exists$ an exact sequence

$$0 \to R \to \mathrm{K}_R(-a) \to C \to 0$$

of graded R-modules s.t. $\mu_R(C) = \mathrm{e}^0_M(C)$.

Notice that

- R is an AGG ring $\implies$ R_M is an AGL ring.
- The converse is not true in general.

Theorem 4.3

Let $R = k[R_1]$ be a CM homogeneous ring with $d = \dim R \geq 1$. Suppose that $|k| = \infty$ and R is not a Gorenstein ring. Then TFAE.

(1) *R is an AGG ring and level.*

(2) *$\mathrm{Q}(R)$ is a Gorenstein ring and $\mathrm{a}(R) = 1 - d$.*

Example 4.4

Let $S = k[X_{ij} \mid 1 \leq i \leq m, 1 \leq j \leq n\ ]$ $(2 \leq m \leq n)$ and k an infinite field. We put

$$R = S/\mathrm{I}_t(X)$$

where $2 \leq t \leq m$, $X = [X_{ij}]$. Then

$$R \text{ is an AGG ring} \iff m = n, \text{ or } m \neq n \text{ and } t = m = 2.$$

Example 4.5

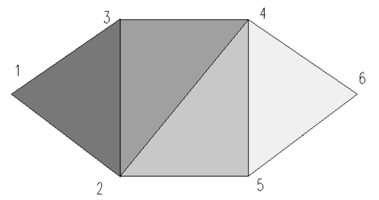
Let $R = k[X_1, X_2, \ldots, X_d]$ $(d \geq 1)$, k an infinite field, and $1 \leq n \in \mathbb{Z}$.

- $R^{(n)} = k[R_n]$ is an AGG ring, if $d \leq 2$.
- Suppose that $d \geq 3$. Then

$$R^{(n)} \text{ is an AGG ring} \iff n \mid d, \text{ or } d = 3 \text{ and } n = 2.$$

Example 4.6

Look at the simplicial complex Δ :



Then $R = k[\Delta]$ is an AGG ring with $\dim R = 3$.

Theorem 4.7

Let $(A, \mathfrak{m})$ be a CM local ring with $|A/\mathfrak{m}| = \infty$, possessing the canonical module K_A. Let I be an $\mathfrak{m}$-primary ideal of A. If $G = \mathrm{gr}_I(A)$ is an AGG ring and $\mathrm{r}(G) = \mathrm{r}(A)$, then A is an AGL ring.

Two-dimensional rational singularities

Setting 5.1

- $(R, \mathfrak{m})$ a CM local ring with $d = \dim R$
- $|R/\mathfrak{m}| = \infty$
- $\exists$ K_R the canonical module of R
- $v(R) = \mu_R(\mathfrak{m})$, $\mathrm{e}(R) = \mathrm{e}^0_{\mathfrak{m}}(R)$
- $G = \mathrm{gr}_{\mathfrak{m}}(R) = \bigoplus_{n \geq 0} \mathfrak{m}^n/\mathfrak{m}^{n+1}$

Theorem 5.2

(1) *Suppose that G is an AGG ring and level. Then R is an AGL ring.*

(2) *Suppose that R is an AGL ring and $v(R) = \mathrm{e}(R) + d - 1$. Then G is an AGG ring and level.*

Corollary 5.3

Suppose that $v(R) = \mathrm{e}(R) + d - 1$. Then TFAE.

(1) *R is an AGL ring.*

(2) *G is an AGG ring.*

(3) *$\mathrm{Q}(G)$ is a Gorenstein ring.*

Corollary 5.4

Suppose that $v(R) = \mathrm{e}(R) + d - 1$ *and* R *is a normal ring. If* $\mathfrak{m}$ *is a normal ideal, then* R *is an AGL ring.*

Corollary 5.5

Every two-dimensional rational singularity is an AGL ring.

Corollary 5.6

Every two-dimensional CM complete local ring R *of finite CM representation type is an AGL ring, provided* R *contains a field of characteristic* 0.

Further results

- [Goto-Matsuoka-T-Yoshida, Goto-Rahimi-T-Truong]

  ⋯ Almost Gorenstein Rees algebras
- [Goto-Takahashi-T]

  ⋯ Almost Gorenstein rings and Ulrich ideals
- [Celikbas-Celikbas-Goto-T]

  ⋯ Almost Gorenstein Arf rings
- [Higashitani]

  ⋯ Almost Gorenstein homogeneous rings and *h*-vectors
- [Miyazaki]

  ⋯ Almost Gorenstein Hibi rings
- [Matsuoka-Murai]

  ⋯ Almost Gorenstein Stanley-Reisner rings

Thank you so much for your attention.

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